Fields Of Electric and Magnetic Charges

Electric Charges

The presence of electric charges gives rise to electric force and induction fields in the space surrounding these charges. A vector electric field **E**(**r**) is said to exist at a spatial point **r** if a vector force **F**(**r**) is exerted on a test electric charge q brought to that point. The electric field **E(r)** is defined as

$$\mathbf{E}(\mathbf{r}) = \lim_{q \to 0} \frac{\mathbf{F}(\mathbf{r})}{q} \; volts / meter$$

and is regarded as characterizing the field at the given point even in the absence of the test charge (The MKS system of units is used). It is experimentally observed that the static electric field is conservative, i.e. the work done on moving a charge around a closed path is zero, whence the integral $\int \mathbf{E} \bullet d\mathbf{s}$ about a closed path vanishes, a fact represented vectorially by $\nabla \times \mathbf{E} = 0$.

A vector electric induction **D**(**r**) is said to exist at a point **r** if an electric charge is induced at that point on a small separable electrically conducting plate of area ΔS whose normal unit vector is **n**. If the maximum charge q_{ind} is induced when the plate is oriented perpendicular to the direction of the unit vector n, the electric induction **D**(**r**) is defined as

$$\mathbf{D}(\mathbf{r}) = \lim_{q_{ind} \to 0} \frac{q_{ind}}{\Delta S} \mathbf{n} \; coulombs / meter^2$$

where the induced charge is readily measured if the plate is separated into its two duplicate parts. From this definition one infers Gauss's law that the surface integral of the normal component of the electric induction is equal to the charge enclosed within the surface, or in integral form:

$$\iint \mathbf{D} \bullet d\mathbf{S} = \iiint \rho \; dV \quad or \quad \nabla \bullet \mathbf{D} = \rho$$

where ρ is the charge density. In a vacuum one finds that **D** and **E** are proportional, i. e.

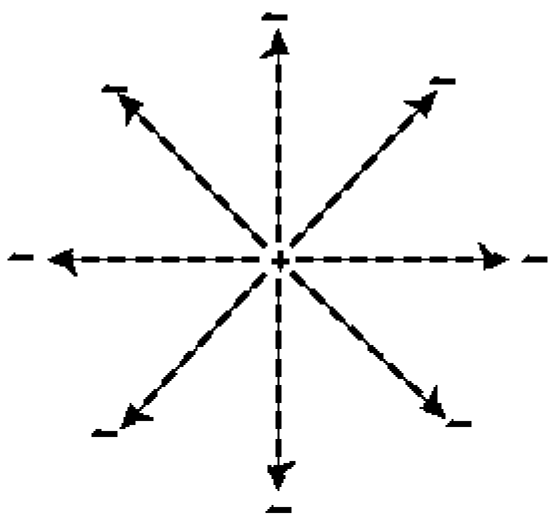
$$\mathbf{D} = \varepsilon_0 \, \mathbf{E}$$

where $\varepsilon_0 = 8.854 \times 10^{-12}$ farads/meter is the vacuum dielectric constant.

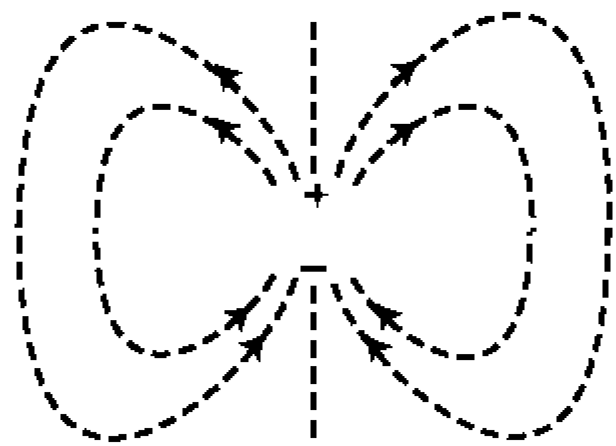
The electric induction **D**(**r**) may be viewed not only as an induced charge per unit area but also as a "vacuum" dipole moment per unit volume. A polar dipole, which is composed of a positive and negative point charge of magnitude *q* separated by a vector distance ***h*,** has a dipole moment *q**h***. A dielectric material composed of atoms or molecules is describable by a real polar dipole moment per unit volume.

In the Faraday view of the electric field between two oppositely charged parallel metallic plates, lines of electric force emanate from the positive charges on one plate and end on the negative charges of the second plate. One set of lines is formed of vacuum dipoles, with a dipole moment ε_oE per unit volume. The other set of lines of force is composed of real polar dipoles that constitute the dielectric medium whose dipole moment per unit volume defines the polarization vector P. The total dipole moment per unit volume, or induced charge per unit area at any transverse surface S within the dielectric is given by $\mathbf{D} = \varepsilon_o \mathbf{E} + \mathbf{P}$. **I**n a linear medium $\mathbf{P} = (\varepsilon - \varepsilon_o)\mathbf{E}$, whence in vacuum **P** evidently vanishes. One further notes that, for a given charge on each of the parallel plates of' a condenser, the ε_o **E** lines (which are in tension) transmit force while the **P** lines absorb it. Since the effect of' the dielectric is to decrease **E**, the attractive force between the two condenser plates is also diminished. Generally, in a linear dielectric medium the force between charges is reduced (as compared to the vacuum case) by a factor $1/\varepsilon$, where ε is the relative electric constant of the medium.

Different charge configurations produce different lines of force field patterns. The fields of a point charge and a point dipole are displayed below.



Field of a Point Charge



Field of a Dipole Charge

More complex quadrupole, octupole, etc., charge structures yield similar but more complex patterns., all characterized by field lines that start on positive charges and terminate on negative charges. Any finitely extended distribution of electric charge may be regarded as a superposition of point, dipole, quadrupole, etc. charges.

Magnetic Charges

Magnetic charges create force and induction fields that are analogous, or dual, to those of electric charges, and hence can be defined in a manner paralleling the discussion of the electric case. Since point magnetic charges and perfect magnetic conductors (dual of metals) are not in a strict sense physically realizable, empirical definitions of force and induction fields are not as clear cut as in the electrical case. Nevertheless, the conceptual advantages of the dualism are such as to make desirable a similar treatment of the magnetic case.

By duality one thus defines a vector magnetic field **H(r)** in terms of the force **F(r)** exerted on a small test magnetic charge q* at a point **r** as

$$\mathbf{H}(\mathbf{r}) = \lim_{q^* \to 0} \frac{\mathbf{F}(\mathbf{r})}{q} \; amps/meter$$

Similarly, the vector magnetic induction **B** is defined in terms of the induced magnetic charge $q^*_{\mathbf{ind}}$ on an area ΔS, with normal **n**, of a magnetic conductor as

$$\mathbf{B}(\mathbf{r}) = \lim_{q^*_{ind} \to 0} \frac{q^*_{ind}}{\Delta S} \mathbf{n} \; webers/meter^2$$

or equivalently, as a fictitious magnetic dipole moment per unit volume. In vacuum one finds that

$$\mathbf{B} = \mu_0 \mathbf{H}$$

where $\mu_0 = 1.257 \times 10^{-7}$ henries/meter is the permeability of vacuum.

Real polar magnetic dipoles exist in nature, for example in ferromagnetic materials. One defines the magnetic polarization **P*** as the real magnetic dipole moment per unit volume. Thus, in a magnetic medium the total magnetic induction is

$$\mathbf{B} = \mu_o \mathbf{H} + \mathbf{P^*}, \quad \text{in a linear medium} \quad \mathbf{P^*} = (\mu - \mu_o)\mathbf{H}.$$

Sometimes the magnetization vector $\mathbf{M} = \mathbf{P}^*/\mu_0$ is employed instead of the polarization $\mathbf{P}^*$.

The Faraday view of a ferromagnetic space between two parallel planes of opposite magnetic charge leads to a line of (magnetic) force picture dual to that for an electric condenser. Likewise, the field patterns of point magnetic charges and dipoles are similar to those of electric charges and dipoles. Fields of finitely extended distributions of magnetic charges can be regarded as a superposition of point, dipole, quadrupole, etc., charge distributions.